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Factor for the Lunar Ephemeris*

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This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.



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# Earth Radius/Kilometer Conversion Factor for the Lunar Ephemeris

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## I. Introduction

IN precision simulations of lunar and interplanetary probe trajectories, the equations of motion contain terms that require a knowledge of the position of the moon at any time during flight. Furthermore, for calculation of the probe's position and velocity relative to the moon, both the position and velocity of the moon are required. Common practice is to obtain the position of the moon from the Lunar Ephemeris<sup>1</sup> and subsequently develop the velocity by numerical differentiation of the position.

A difficulty arises in that the lunar coordinates given in the Ephemeris use the earth radius as a unit of length, whereas, for practical reasons, a laboratory unit of length such as the kilometer is employed as the basic unit of measure in trajectory calculations. The problem is then one of determining a conversion (or scale) factor to convert the lunar coordinates from earth radii to kilometers.

At first impulse, one would choose the best available value of the earth equatorial radius, as expressed in kilometers, and use this as the conversion factor. However, to do so would be incorrect. Rather, the conversion factor must be computed from a relationship that is a function of the moon's mean motion and the gravitational constants of the earth and moon.

It is the purpose of this paper to develop this relationship and give a value for the earth radius/kilometer conversion factor for the Lunar Ephemeris.

## II. Analysis

De Sitter<sup>2</sup> defines the sine of the mean equatorial lunar parallax as

$$\sin \pi_c = b/a \quad (1)$$

where  $b$  is the equatorial radius of the earth and  $a$  is the "constant of the moon's variation orbit" defined in Brown's theory.<sup>3, 4</sup> The constant  $\alpha$  is defined by the relation

$$n^2 \alpha^3 = GM_E + GM_M \quad (2)$$

where  $G$  is the universal gravitational constant,  $M_E$  and  $M_M$  are the masses of the earth and moon, and  $n$  is the moon's mean sidereal motion, which is taken as a fundamental invariant, the value of which, as used by Brown, is  $n = 0.2661699563 \times 10^{-5}$  rad/sec. Brown also gives a relation between  $a$  and  $\alpha$ ; it is

$$a/\alpha = 0.999093141975298 \quad (3)$$

Thus, it is seen that, if  $GM_E + GM_M$  and  $\sin \pi_c$  are known, then  $b$  can be calculated. Brown gives

$$\sin \pi_c = 3422''.54 = 0.01659294212$$

Thus,

$$b/a = 0.01659294212$$

or

$$a = 60.2665876b \quad (4)$$

which establishes the relation between the mean lunar distance and earth radius as Brown sees it. From Eq. (3),

$$\alpha = 1.000907681a \quad (5)$$

Then

$$\alpha = 60.32129044b \quad (6)$$

After substituting (6) into (2), we obtain

$$b = 0.0165778946[(GM_E + GM_M)/n^2]^{1/3} \quad (7)$$

or finally

$$b = 86.315745(GM_E + GM_M)^{1/3} \quad (8)$$

## III. Conclusion

The formula, Eq. (8), is the relation mentioned earlier for computing the earth radius/kilometer conversion factor  $b$ , when  $GM_E$  and  $GM_M$  are given in kilometers cubed per seconds squared. This relation is analogous to Kepler's third law and must be maintained. If some other value of  $b$  is used, the well-determined mean motion of the moon is not preserved.

To calculate a value of  $b$  from Eq. (8), let  $GM_E = 398603.2$  km<sup>3</sup>/sec<sup>2</sup> and note that  $K_{ME}$ , "the coefficient of the indirect acceleration of the moon on the earth," has been well determined from the orbit of Mariner II,<sup>6</sup> where

$$K_{ME} = GM_M/b^2 = 1.205116 \pm 0.000049 \times 10^{-4} \quad (9)$$

A cubic equation in  $GM_M$  can be formed by using Eq. (8) and (9) to eliminate  $b$  and yield

$$GM_M = 4902.78 \pm 0.20 \text{ km}^3/\text{sec}^2$$

and thus give an earth-moon mass ratio of

$$\mu = 81.3015 \pm 0.0033$$

Subsequently, from Eq. (9),

$$b = (GM_M/K_{ME})^{1/2} \quad (10)$$

or, finally, the numerical value of the earth radius/kilometer conversion factor is

$$b = 6378.3255 \text{ km}$$

Upon substituting this into Eq. (4), we obtain the mean distance of the moon

$$a = 384399.9 \text{ km}$$

which compares favorably with Fischer's<sup>7</sup> value of 384,400 km and the radar-determined value of 384,400.2 km.<sup>8</sup>

To conclude, it is important to realize that the value of earth radius  $b$  is not the same as the actual radius of the earth; it is merely the conversion factor and is used *only* for scaling the Lunar Ephemeris from earth radii to kilometers. The value of the actual radius of the earth  $R_E$  is taken to be  $R_E = 6378.165$  km.<sup>5</sup>

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## References

- <sup>1</sup> Block, N., "Computation of lunar positions from the improved Brown lunar theory," Research Summary 36-12, Vol. 1, pp. 125-128, Jet Propulsion Lab., Pasadena, Calif. (January 2, 1962).
- <sup>2</sup> De Sitter, W., "On the system of astronomical constants," Bull. Astron. Inst. Netherlands **VIII**, 213-231 (July 8, 1938).
- <sup>3</sup> Brown, E. W., "Theory of the motion of the moon; containing a new calculation of the expressions for the coordinates of the moon in terms of the time," *Memoirs of the Royal Astronomical Society* (Royal Astronomical Society, London, 1896-1899), Vol. **LIII**, Part 1, Chaps. I-IV, pp. 39-117.
- <sup>4</sup> Brouwer, D. and Clemence, G. M., *Methods of Celestial Mechanics* (Academic Press, New York and London, 1961), p. 347.
- <sup>5</sup> Clarke, V. C., Jr., "Constants and related data used in trajectory calculations at the Jet Propulsion Laboratory," TR32-273, Jet Propulsion Lab., Pasadena, Calif. (May 1, 1962).
- <sup>6</sup> Anderson, J. D. and Null, G., "The evaluation of certain physical constants from the radio tracking of Mariner II," AIAA Preprint 63-424 (August 1963).
- <sup>7</sup> Fischer, I., "Parallax of the moon in terms of a world geodetic system," Astron. J. **67**, 373-378 (1962).
- <sup>8</sup> Yaplee, B. S., Knowles, S. H., Shapiro, A., Craig, K. J., and Brouwer, D., "The mean distance to the moon as determined by radar," Intern. Astron. Union Symposium 21, System of Astronomical Constants, Paris, France (May 27-31, 1963).